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# EFFECT OF FREE STREAM TURBULENCE ON THE TURBULENT BOUNDARY LAYER

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**Abstract**—It is pointed out in the present study that by allowing the entrainment of free stream turbulence into the turbulent boundary layer and performing an overall turbulent energy balance, very satisfactory quantitative predictions of the effect of free stream turbulence upon the turbulent boundary layer behavior can be made. Furthermore, both theory and experiment indicate a 30 per cent increase in wall friction arising from a free stream turbulence level of only 5 per cent. Since it is felt that some sort of Reynolds analogy would hold, it is suggested that the frequently quoted result of little direct effect of free stream turbulence level on the heat transfer other than a movement of the transition point may be valid only at low Reynolds numbers.

## NOMENCLATURE

$a_n$, structural coefficients of turbulence;
$C_p$, specific heat;
$\mathscr{D}$, sublayer damping factor;
$E$, turbulent energy entrainment (see equation (10));
$H_1$, ratio of displacement thickness to boundary-layer thickness;
$k$, thermal conductivity;
$k_T$, turbulent thermal conductivity;
$L$, dissipation length;
$L_\infty$, wake value of dissipation length;
$\ell$, mixing length;
$\ell_\infty$, wake value of mixing length;
$P$, pressure and probability function;
$Pr$, Prandtl number;
$Pr_T$, turbulent Prandtl number;
$Q$, heat flux;
$\bar{q}^2$, turbulence kinetic energy;
$R_\theta$, Reynolds number based upon momentum thickness;
$R_\tau$, turbulence Reynolds number;
$r$, radius;
$T$, static temperature;
$T_u$, free stream turbulence intensity;
$T_w$, wall temperature;
$T^\circ$, total temperature;
$u$, streamwise velocity;
$v$, transverse velocity;
$w$, cross flow velocity;
$x$, streamwise coordinate;
$y$, transverse coordinate;
$y^+$, dimensionless transverse coordinate.

### Greek letters

$\alpha$, indicator equal to one for axisymmetric flow, zero for two-dimensional flow;
$\Gamma$, intermittency factor;
$\delta$, boundary-layer thickness;
$\delta^+$, reference length;
$\delta^*$, displacement thickness;
$\delta_S$, sublayer thickness;
$\delta_{0.99}$, boundary-layer thickness where $u = 0.99\,u_e$;
$\varepsilon$, turbulence dissipation;
$\eta$, dimensionless transverse coordinate;
$\theta$, momentum thickness;
$\kappa$, von Karman constant;
$\mu$, viscosity;
$\nu$, kinematic viscosity;
$\nu_T$, kinematic eddy viscosity;
$\rho$, density;
$\tau$, shear stress;
$\phi_1, \phi_2, \phi_3$, integral functions [see equations (13) through (15)].

### Subscripts

$e$, boundary-layer edge condition;
$\infty$, free stream condition;
$w$, wall condition.

### Superscripts

$\bar{\ }$, average quantity;
$'$, fluctuating quantity.

## 1. INTRODUCTION

IN RECENT years there has been considerable progress in the development of both incompressible and compressible turbulent boundary-layer prediction methods (see, for instance [1–3]). However, most of the boundary-layer calculation methods presented to date, with at least the exception of the procedure of McDonald and Fish [1], have ignored the effect of free stream turbulence upon the turbulent boundary-layer development in spite of the fact that this effect is known to be appreciable in certain cases [4]. In addition, as Schilichting and Das [5] point out, the level of free stream turbulence encountered in typical turbomachinery applications, a case of particular interest to the present authors, is such as to cause very substantial effects on the turbulent boundary-layer development. Recently, Huffman *et al.* [6] and Charnay *et al.* [7] have measured in detail the effect of free stream turbulence upon both the mean and fluctuating components of velocity within a turbulent boundary. These measurements show very clearly the extremely large effect of free stream turbulence upon the turbulent transport in the outer region of the boundary layer, Huffman *et al.*, observing as much as six-fold increase in the conventionally defined Prandtl's mixing length in the center region of the boundary layer with about 5 per cent free stream turbulence. The observations of Huffman *et al.* [6] and Charnay *et al.* [7] are in close agreement with the findings of Kline *et al.* [4]. Kline *et al.* found experimentally that, with free stream turbulence levels on the order of 5 per cent or more, very large changes occurred in both the growth and shape of the mean velocity profile in a turbulent boundary layer, effects similar to those observed by Huffman *et al.* and Charnay *et al.* and consistent with a marked increase in turbulent transport within the boundary layer.

It has, of course, long been recognized that increasing the free stream turbulence level can cause a forward shift of the transition region from laminar to turbulent flow [8]. It seems generally agreed [9] that on the basis of presently available experimental evidence the forward shift of the transition point is the only factor influencing the heat-transfer rate to a turbulent boundary layer with varying free stream turbulence level. This is at first sight somewhat surprising in view of the marked increase in turbulent transport consistently found, directly or indirectly, by investigators studying the velocity profile development under the influence of varying free stream turbulence levels. It is possible that, since heat transfer is a wall dominated phenomena, the large increase in turbulent transport occurring in the outer region of the boundary layer would not so directly affect the heat transfer. This is to some extent quite plausible, but on the basis of the measurements of Huffman *et al.* and Charnay *et al.*, both of whom found an effect on wall friction, one would expect that at least some of the effect would be felt on the heat transfer at the wall. The aim of the present study was, therefore, twofold: firstly, to see if the procedure of McDonald and Fish [1] could give quantitatively accurate predictions of the effect of free stream turbulence on the turbulent boundary-layer development; and secondly, to see if any light could be shed on the apparently anomalous observation of little direct effect of free stream turbulence on the turbulent boundary-layer heat-transfer rate. The problem of the effect of free stream turbulence at a front stagnation point [9] is not considered in the present study.

## 2. THEORY

*The basic equations*

Within the framework of the usual boundary-layer approximations, various authors, for example, Schubauer and Tchen [10], have reduced the time-averaged Navier–Stokes equations to the compressible boundary-layer equations of motion. In the boundary-layer equations, it is convenient to represent the turbulent stress contribution to the total shear stress, $\tau$, in terms of an effective turbulent viscosity, $\nu_T$, and the turbulent temperature correlation contribution to the total heat flux, $Q$, in terms of an effective turbulent conductivity, $k_T$, where

$$\bar{\rho}\nu_T\partial\bar{u}/\partial y = -\bar{\rho}\overline{u'v'};\quad k_T\partial\bar{T}/\partial y = -\bar{\rho}C_p\overline{v'T'}. \tag{1}$$

The prime denotes a fluctuating quantity found in turbulent flow and the bar denotes a time-mean average. The effective turbulent conductivity is now related to the effective turbulent viscosity by the introduction of the turbulent Prandtl number defined by

$$Pr_T = C_p\bar{\rho}\nu_T/k_T. \tag{2}$$

When the turbulent and molecular Prandtl numbers are introduced and, in addition, the usual assumptions are made that the contributions from the longitudinal gradient of the Reynolds normal stress and normal pressure gradients are negligible, then for steady two-dimensional flow the boundary-layer equations, together with the continuity equation, may be written in the form

$$\bar{\rho}\bar{u}\frac{\partial\bar{u}}{\partial x}+\overline{\rho v}\frac{\partial\bar{u}}{\partial y} = -\frac{\mathrm{d}p}{\mathrm{d}x}+\frac{\partial}{\partial y}\left\{(\bar{\mu}+\bar{\rho}\nu_T)\frac{\partial\bar{u}}{\partial y}\right\} \tag{3}$$

$$\bar{\rho}\bar{u}C_p\frac{\partial\bar{T}^\circ}{\partial x}+\overline{\rho v}C_p\frac{\partial\bar{T}^\circ}{\partial y} = \frac{\partial}{\partial y}\left\{\left(\frac{\bar{\mu}}{Pr}+\frac{\bar{\rho}\nu_T}{Pr_T}\right)C_p\frac{\partial\bar{T}^\circ}{\partial y}\right\} + \frac{\partial}{\partial y}\left\{\left[\left(1-\frac{1}{Pr}\right)\bar{\mu}+\left(1-\frac{1}{Pr_T}\right)\bar{\rho}\nu_T\right]\bar{u}\frac{\partial\bar{u}}{\partial y}\right\} \tag{4}$$

$$\frac{\partial\bar{\rho}\bar{u}r^\alpha}{\partial x}+\frac{\partial\overline{\rho v}r^\alpha}{\partial y} = 0 \tag{5}$$

where the stagnation temperature $\bar{T}^{\circ}$, total apparent stress $\bar{\tau}$, and total effective heat flux $Q$, are defined as

$$T^{\circ} = T + \frac{u^2}{2C_p}, \quad \tau = \bar{\mu}\frac{\partial \bar{u}}{\partial y} - \bar{\rho}\overline{u'v'},$$

$$Q = \bar{k}\frac{\partial \bar{T}}{\partial y} - \bar{\rho}C_p\overline{v'T'}. \quad (6)$$

These equations are, of course, also valid for laminar flow when all the turbulent correlations are zero. For two-dimensional flow $\alpha$ is zero, for axisymmetric flow $\alpha = 1$.

The wall and free stream boundary conditions employed in the solution are

$$y = 0 \quad \overline{\rho v} = (\overline{\rho v})_w, \ \bar{T}^{\circ} = T_w \quad \text{or} \quad \partial \bar{T}/\partial y = 0$$
$$y \to \infty \quad \bar{\rho}\bar{u} = \bar{\rho}_e \bar{u}_e, \ \bar{T}^{\circ} = \bar{T}^{\circ}_e, \quad (7)$$
$$\partial \bar{u}/\partial y = 0, \ \partial \bar{T}^{\circ}/\partial y = 0$$

where the subscripts $w$ and $e$ denote the wall and free stream values, respectively.

In order to predict the development of the mean velocity and temperature field it remains to specify the effective turbulent viscosity and turbulent Prandtl number in terms of the mean flow variables and this is described in the following section. The term $\overline{\rho v}$ is eliminated from the momentum and energy equation by application of the continuity equation and, as indicated in [1], upon specification of the turbulent transport coefficients, the resulting equations are solved by a finite-difference procedure.

*The turbulence model*

The turbulence model originally presented by McDonald and Camarata [11] for two-dimensional incompressible flow, forms the basis for the McDonald–Fish [1] boundary-layer procedure and, therefore, at this point it is useful to describe the model in some detail. The turbulence model, developed in detail in [1] and [11], is based upon a solution of the turbulence kinetic energy equation. The turbulence kinetic energy equation is a conservation equation derived from the Navier–Stokes equations by writing the instantaneous quantities as a sum of mean and fluctuating parts. The $i$th Navier–Stokes momentum conservation equation ($i = 1, 2, 3$, referring to the three coordinate directions) is multiplied by the $i$th component of fluctuating velocity and the average of the resulting three equations is taken. The three averaged equations are summed to obtain the turbulence kinetic energy equation. A derivation and discussion of the turbulence kinetic energy equation for compressible flow is given by Shamroth and McDonald [12].

As shown in [12], the boundary-layer approximation to the compressible turbulence kinetic energy equation is given by

$$\underbrace{\frac{\partial}{\partial x}(\tfrac{1}{2}\bar{\rho}\bar{u}\bar{q}^2) + \frac{\partial}{\partial y}(\tfrac{1}{2}\overline{\rho v}\bar{q}^2)}_{\text{advection}} = \underbrace{-\bar{\rho}\overline{u'v'}\frac{\partial \bar{u}}{\partial y}}_{\text{production}}$$
$$\underbrace{-\frac{\partial}{\partial y}(\overline{P'v'} + \tfrac{1}{2}\overline{(\rho v)'q^2})}_{\text{diffusion}} \underbrace{- \bar{\rho}\varepsilon}_{\text{dissipation}} \quad (8)$$
$$\underbrace{-\bar{\rho}(\bar{u}'^2 - \bar{v}'^2)\frac{\partial \bar{u}}{\partial x}}_{\text{normal stress production}} + \underbrace{\overline{P'\frac{\partial u_i'}{\partial x_i}}}_{\text{pressure–dilitation}}.$$

All calculations reported in the present study were made with the usual assumption of zero-dilitation contribution to the energy balance. The turbulence model is developed by integrating equation (8) with respect to $y$ between the limits $y = 0$ and $y = \delta$ which leads to

$$\tfrac{1}{2}\frac{d}{dx}\int_0^{\delta} \bar{\rho}\bar{u}\bar{q}^2\,dy = \int_0^{\delta} -\bar{\rho}\overline{u'v'}\frac{\partial \bar{u}}{\partial y}\,dy - \int_0^{\delta} \bar{\rho}\varepsilon\,dy$$
$$- \int_0^{\delta} \bar{\rho}(\bar{u}'^2 - \bar{v}'^2)\frac{\partial \bar{u}}{\partial x}\,dy + \int_0^{\delta} \overline{P'\frac{\partial u_i'}{\partial x_i}}\,dy + E \quad (9)$$

where

$$E = \left[\tfrac{1}{2}\bar{q}^2\left(\bar{\rho}\bar{u}\frac{\partial \delta}{\partial x} - \overline{\rho v}\right) - \overline{P'v'} + \tfrac{1}{2}\overline{(\rho v)'q^2}\right]_e \quad (10)$$

and it is noted that $E$ represents the turbulent energy entrained by the boundary layer from the free stream. Thus the present formulation explicitly allows for the presence of free stream turbulence. Structural coefficients $a_n$ and $L$ are introduced, together with a mixing length $\ell$ defined as

$$\left(\bar{q}^2 - f\left(\frac{y}{\delta}\right)\bar{q}_e^2\right)a_1 = -\overline{u'v'}, \quad \bar{u}'^2 = a_2\bar{q}^2,$$
$$\bar{v}'^2 = a_3\bar{q}^2, \quad \overline{w'^2} = (1 - a_2 - a_3)\bar{q}^2$$
$$\varepsilon = (-\overline{u'v'})^{3/2}/L, \quad (-\overline{u'v'})^{1/2} = \ell\frac{\partial \bar{u}}{\partial y} \quad (11)$$
$$f\left(\frac{y}{\delta}\right) = (1 - \cos \pi y/\delta)/2$$

where the only departure from [1] and [11] is the inclusion of a free stream turbulence contribution $f(y/\delta)\bar{q}_e^2$ to the turbulence kinetic energy within the boundary layer. Normally in laboratory boundary layers this contribution can be neglected but is included for completeness in the present study since large values of $\bar{q}_e^2$ are to be explored. It is noted that although the contribution $f(y/\delta)\bar{q}_e^2$ to the shear intensity relationship was neglected in [10], as being negligible for the cases considered therein, the source term $E$, representing the entrainment of free stream turbulence in the turbulence kinetic energy equation, was retained in [10], thus allowing a direct effect of free stream turbulence upon the Reynolds shear stress level to be predicted. For fully-developed turbulence the structural coefficients $a_1$, $a_2$ and $a_3$ are assumed constant having values 0·15, 0·50 and 0·20, respectively. As is discussed in [1], $a_1$ is treated as a Reynolds number dependent in the traditional and low Reynolds number regime. Using equation (11), equation (9) is put in the form

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\phi_1\rho_e u_e^3\delta^+}{2a_1}\right) = \rho_e u_e^3\left(\phi_2-\phi_3+\frac{E}{\rho_e u_e^3}\right) \quad (12)$$

where

$$\phi_1 = \int_0^{\delta/\delta^+} \frac{\bar{\rho}\bar{u}}{\rho_e u_e}\left\{\left(\frac{\ell}{\delta^+}\frac{\partial\bar{u}/u_e}{\partial\eta}\right)+a_1 f(y/\delta)\frac{\bar{q}_e^2}{u_e^2}\right\}\mathrm{d}\eta \quad (13)$$

$$\phi_2 = \int_0^{\delta/\delta^+} \frac{\bar{\rho}}{\rho_e}\left(\frac{\ell}{\delta^+}\right)^2\left(\frac{\partial\bar{u}/u_e}{\partial\eta}\right)^3\left(1-\frac{\ell}{L}\right)\mathrm{d}\eta \quad (14)$$

$$\phi_3 = \int_0^{\delta/\delta^+} \frac{\bar{\rho}}{\rho_e}\left(\frac{a_2-a_3}{a_1}\right)\left(\left\{\frac{\ell}{\delta^+}\frac{\partial\bar{u}/u_e}{\partial\eta}\right\} + a_1 f(y/\delta)\frac{\bar{q}_e^2}{u_e^2}\right)\frac{\delta^+}{u_e}\frac{\partial\bar{u}}{\partial x_y}\mathrm{d}\eta \quad (15)$$

where $\eta$ is a nondimensionalized transverse distance $y/\delta^+$, $\delta^+$ is an arbitrary reference length, and $\delta$ the boundary-layer thickness. Once again, only in contribution from the free stream turbulence to the shear–intensity relationship, $f(y/\delta)\bar{q}_e^2$, does the foregoing differ from [1] and [11]. Insofar as the thickness $\delta$ is concerned, this is taken as the point where the stress $-\overline{u'v'}$ had fallen to 0·1 per cent of its maximum value to ensure a complete boundary-layer energy balance. The thickness at the point where the velocity achieves 0·99 of free stream is designated $\delta_{0\cdot 99}$ and this was generally taken between 0 and 50 per cent smaller than the stress thickness $\delta$. The contribution to the energy balance arising from the region between $\delta$ and $\delta_{0\cdot 99}$ was usually small but taken into account for completeness.

The l.h.s. of equation (12) represents the streamwise rate of change of turbulence kinetic energy and is derived directly from the turbulence kinetic energy advection term. The term $\rho_e u_e^3\phi_2$ represents the integral of turbulence production minus dissipation and $\rho_e u_e^3\phi_3$ is the normal stress production..The terms designated by $E$ are turbulent source terms resulting from energy imparted to the boundary layer by the free stream. As can be seen from equation (10), $E$ is the sum of two major contributions, the first $(q^2/2)(\bar{\rho}\bar{u}\partial\delta/\partial x-\bar{\rho}\bar{v})_e$ represents the free stream velocity disturbance (i.e. free stream turbulence entrained by the boundary layer) and the second, $\overline{P'v'}+\overline{(\rho v)'q^2/2}$, represents the direct absorption of acoustic energy. (This acoustic energy absorption term will be subsequently assumed to be negligible.)

For fully-developed turbulent flow, as in [1], $L$ and $\ell$ are given by

$$\frac{L}{\delta_{0\cdot 99}} = 0\cdot 1\tanh[\kappa y/(0\cdot 1\,\delta_{0\cdot 99})] \quad (16)$$

$$\frac{\ell}{\delta_{0\cdot 99}} = \frac{\ell_\infty}{\delta_{0\cdot 99}}\left\{\tanh\left(\frac{\kappa y}{\ell_\infty}\right) + \left(1-\tanh\frac{\kappa\delta_{0\cdot 99}}{\ell_\infty}\right)\left(\frac{1-\cos\dfrac{\pi y}{\delta_{0\cdot 99}}}{2}\right)\right\} \quad (17)$$

where $\ell_\infty$ is the "wake" value of the mixing length at any particular streamwise station. In the earlier work of [1] and [11] the cosine term in the assumed mixing length profile was not present. Thus in the earlier work of [1] and [11] the mixing length $\ell$ could never exceed $\kappa y$, a restriction which never significantly constrained the previous prediction. However, in trying to predict the effect of some high free stream turbulence levels it became evident that both theory and experiment indicated mixing lengths greater than $\kappa y$. Consequently, the assumed profile family was enlarged to allow a gradual cosine contribution to the profile to appear for large values of the mixing length. Note that the assumed profile family is still a single parameter profile with $\ell_\infty$ as a scale and that for conventional values of mixing length in the region $\ell_\infty = 0\cdot 1\,\delta_{0\cdot 99}$, the cosine term does not contribute significantly. Although equations (16) and (17) can give quite fair representations of $\ell$ and $L$ through most of the turbulent boundary layer, it is well-known that they overestimate the length scales within the viscous sublayer and that equation (16) for $L$ is somewhat inaccurate at low Reynolds numbers. Following McDonald and Fish [1] the experimentally observed damping effect in the viscous sublayer is modelled by assuming intermittent turbulence within the sublayer leading to the relation

$$-\overline{u'v'} = \Gamma(-\overline{u'v'})_T = \Gamma(\ell\partial\bar{u}/\partial y)_T^2 = \left(\mathscr{D}\ell_T\frac{\partial\bar{u}}{\partial y}\right)^2 \quad (18)$$

where it is assumed that $\partial u/\partial y$ is the same within and without the turbulent bursts. In equation (18), $\Gamma$ is the intermittency factor, $\mathscr{D}$ the damping factor, and the subscript $T$ indicates the value within the turbulent flow. Obviously, $\mathscr{D}$ is equal to the square root of $\Gamma$. As in [1], the present investigation assumes that the intermittency distributes normally about a mean height $y^+(y^+ = y\sqrt{\tau/\rho}/\nu)$ with a standard deviation $\sigma$ leading to the equation

$$\mathscr{D} = P^{1/2}\{(y^+-\bar{y}^+)/\sigma\} \quad (19)$$

where $P$ is the normal probability function: $\bar{y}^+$ is taken as 23, and $\sigma$ as 8. A detailed discussion of the sublayer damping treatment is presented in [1]. In the present calculations the von Karman constant $\kappa$ was taken to be 0·43.

*Low Reynolds number effects*

In regard to the low Reynolds number effects, Coles [13] has observed and correlated the departure of the mean velocity profile of a flat plate turbulent boundary layer from the usual similarity laws known to hold at higher Reynolds numbers. Using Coles' correlation of the mean velocity profile in the low Reynolds number regime, McDonald [14] integrated the boundary-layer equations of mean motion to obtain local distributions of turbulent shear stress and evaluated the local mixing length distributions from the assumed mean velocity distribution and the computed shear stress distributions. Based upon these calculations, a low Reynolds number correction for the dissipation length of the form

$$L = L_\infty[1+\exp(-1{\cdot}63\ln R_\theta + 9{\cdot}7)] \qquad (20)$$

was derived where $L_\infty/\delta_{0\cdot 99}$ is given by equation (16). In the calculations presented in the present report the dissipation length used was obtained by multiplying equation (20) by the sublayer damping factor, $\mathscr{D}$.

As a result of their work, McDonald and Fish [1] concluded that the structural coefficient $a_1$ also was exhibiting a low Reynolds number effect. The relationship between $a_1$ and viscosity was quantified by introducing a turbulent Reynolds number, $R_\tau$, where the velocity scale is taken as $(-\overline{u'v'})^{1/2}$ and the length scale is taken as the mixing length $\ell$. Upon introduction of the conventional definition of eddy kinematic viscosity, the turbulence Reynolds number $R_\tau$ can be written

$$R_\tau = \frac{\nu_T}{\nu}.$$

To be consistent with the integral turbulence kinetic energy equation, equation (12), a layer-averaged turbulence Reynolds number, $\bar{R}_\tau$, is introduced as

$$\bar{R}_\tau = \frac{1}{\delta}\int_0^\delta \nu_T\,\mathrm{d}y \Bigg/ \frac{1}{\delta_S}\int_0^{\delta_S} \nu\,\mathrm{d}y \qquad (21)$$

where $\delta_S$, the sublayer thickness, is defined as the location at which the laminar stress has fallen to 4 per cent of the total stress (the 4 per cent definition gave a sublayer mean temperature in very good agreement with the so-called Eckert reference temperature).

The McDonald–Fish model assumes that the turbulence Reynolds number, $\bar{R}_\tau$, is the sole variable influencing the development of $a_1$ and a relationship between $a_1$ and $\bar{R}_\tau$ is obtained by considering the development of an incompressible constant pressure flat plate equilibrium turbulent boundary layer. It should be noted that under the assumption that $a_1$ is solely dependent upon $\bar{R}_\tau$ it is only necessary to derive a relation for one set of flow conditions to obtain a universally valid relationship. Using similarity arguments, McDonald and Fish [1] suggested that $a_1$ be determined from the relationship

$$a_1 = a_0(R_\theta/R_{\theta_0})/[1+6{\cdot}666\,a_0(R_\theta/R_{\theta_0}-1)] \qquad (22)$$

where the arbitrary constant $a_0$ is the value of $a_1$ when $R_\theta$ is equal to $R_{\theta_0}$. It should be pointed out that at large values of $R_\theta/R_{\theta_0}$, $a_1$ asymptotes to the fully-developed value of 0·15. The independent variable of equations (22) and (20) is changed from $R_\theta$ to $\bar{R}_\tau$ by using the profile of Maise and McDonald [15] which integrates to give

$$R_\theta = 68{\cdot}1\,\bar{R}_\tau + 614{\cdot}3 \qquad R_\tau > 40. \qquad (23)$$

At low Reynolds numbers good results are obtained using the equation

$$R_\theta = 100\,\bar{R}_\tau^{0\cdot 22} \qquad \bar{R}_\tau \leqslant 1. \qquad (24)$$

In the intermediate range $1 < \bar{R}_\tau < 40$, the two distributions, equations (23) and (24), were joined by a cubic constructed to match the value and slope at the join points. Finally, the constant of integration, $a_0$, is determined on the basis of comparison between experiment and theory. Best agreement between theory and experiment for low Mach number, adiabatic wall boundary layers was obtained by setting $a_0$ equal to 0·0115 when $\bar{R}_\tau$ is equal to unity. Then, from equation (24), $R_{\theta_0} = 100$.

In addition to including the turbulence kinetic energy equation in the set of equations governing the boundary-layer development it is necessary to specify a model for turbulent heat flux contribution, $-\rho Cp\overline{v'T'}$. As previously stated, in the present procedure, $\overline{v'T'}$ is specified by assuming a turbulent Prandtl number, $Pr_T$, which relates the velocity–temperature correlation, $\overline{v'T'}$, to the Reynolds stress, $-\overline{u'v'}$, through equation (2). The turbulent Prandtl number distribution used in the present procedure varies with distance from wall in the manner suggested by Meier and Rotta [16]. At this juncture it should be pointed out that an alternative procedure can be used to determine $\overline{v'T'}$, based upon an easily derived conservation equation for either the quantity $\overline{T'^2}$ or the correlation, $\overline{v'T'}$, which is similar in form to the turbulence kinetic energy equation, equation (8). However, to solve this new conservation equation it is necessary to assume a universal structure relating quantities analogous to dissipation, production, etc. While sufficient experimental data exists to allow valid modeling of the required terms for the turbulence kinetic energy equation, the existing data does not indicate how proper modeling should be carried out for the $\overline{v'T'}$ conservation equation. Thus, at least for the present, the approach based upon a turbulent Prandtl number appears preferable to an approach based upon the $\overline{v'T'}$ conservation equation.

When numerical values of the structural coefficient $a_n$ are specified, equations (17), (19), and (20) are used to represent $L$ and $\ell$, and the pressure dilitation is neglected, the turbulent energy equation, equation (12), becomes an ordinary differential equation with the dependent parameter $\ell_\infty(x)$ which is solved in conjunction with the boundary-layer momentum and energy equations to predict the development of both the mean flow and the turbulent shear stress.

*Preliminary observations based on profile similarity*

In an effort to provide some insight into the system of equations prior to performing the numerical integration, the integrated turbulence kinetic energy equation, equation (12), was further simplified and an equilibrium solution sought for constant streamwise pressure. First of all, the normal stress production integral, $\phi_3$, was neglected, and based on prior experiences this seemed a very reasonable assumption for fully-turbulent boundary layers far from separation. Next, it was supposed that under the action of the free stream turbulence some equilibrium state had been achieved and the normalized convection thickness was not changing appreciably in the streamwise direction. If the Reynolds number based on momentum thickness is greater than say 4000, the coefficient $a_1$ may also be regarded as invarient and so for a constant streamwise pressure the turbulence energy equation reduces to

$$\frac{\phi_1}{2a_1}\frac{d\delta^+}{dx} = \phi_2 + \frac{E}{\rho_e u_e^2} \tag{25}$$

where $\delta^+$ is a thickness scale which, for convenience, is taken to be the boundary-layer thickness $\delta$. Recalling the definitions of the normalized convection and net production integral thickness, $\phi_1$ and $\phi_2$, it now is supposed that the scales $\ell$ and $1-\ell/L$ may be removed from under the integral sign and approximated by a layer-averaged value $(\bar{\ell}/\delta)$ and $(1-\bar{\ell}/\bar{L})$, thus giving

$$\phi_1 = (\bar{\ell}/\delta)^2\bar{\phi}_1 \tag{26}$$

$$\phi_2 = (\bar{\ell}/\delta)^2(1-\bar{\ell}/\bar{L})\bar{\phi}_2. \tag{27}$$

where $\bar{\ell}$ and $\bar{L}$ are probably representative of the nearly constant values of $\ell$ and $L$ pertaining in the outer region of the boundary layer. The bars denote parameters which are now only a function of the mean profile and for the present simplified analysis we have neglected the direct contribution of $\bar{q}_e^2$ to $\phi_1$ and $\phi_2$ as being negligible. However, in the source term $E/\rho_e u_e^3$, $(\bar{q}_e^2/U_e^2)$ is equated to $3T_u^2$ where $T_u = (\bar{u}'^2)_e^{1/2}/U_e$ and isotropic free stream turbulence is assumed. In this way equation (25) can be written as

$$\left(\frac{\bar{\ell}}{\delta}\right)^2\frac{\bar{\phi}_1}{2a_1}\frac{d\delta}{dx} = \left(\frac{\bar{\ell}}{\delta}\right)^2\left(1-\frac{\bar{\ell}}{\bar{L}}\right)\bar{\phi}_2 + \tfrac{3}{2}T_u^2\frac{d\delta}{dx}(1-H_1) \tag{28}$$

since for a constant streamwise pressure boundary layer the integrated continuity equation yields

$$\left(\frac{v}{u}\right)_e = \frac{d\delta^*}{dx} = H_1\frac{d\delta}{dx} \tag{29}$$

where $H_1 = \delta^*/\delta$ and for the equilibrium conditions considered $H_1$ does not vary appreciably with $x$. If now attention is restricted to cases where $(\bar{\ell}/\bar{L})$ is near unity, the very simple relationship between free stream turbulence and average mixing length is obtained that

$$\frac{\bar{\ell}}{\delta} = \left[\frac{3a_1(1-H_1)}{\bar{\phi}_1}\right]^{1/2} T_u \tag{30}$$

and for simple power law velocity profiles of the type $u/u_e = (y/\delta)^n$ it can readily be ascertained that $\bar{\phi}_1 = n/2$ and $H_1 = n/(n+1)$ so that for a power law profile with $a_1$ taking its accepted value of 0·15, equation (30) can be written

$$\frac{\bar{\ell}}{\delta} = [0{\cdot}9n(n+1)]^{-1/2}T_u \tag{31}$$

and for the usual $n = 1/7$ power profile the result is that only a 3 per cent free stream turbulent level is required to maintain an average mixing length $\bar{\ell}$ of $0{\cdot}1\delta$ in a flat plate boundary layer, a value about equal to the dissipation length $\bar{L}$ in keeping with the assumption that $\bar{\ell}/\bar{L} \approx 1$, and about 10 per cent higher than the usual value for the outer region mixing length of such a boundary layer. Secondly, it is noted that the outer region mixing length varies linearly with free stream turbulence level when the shape of the mean velocity profile is fixed. However, Charnay *et al.* [7] showed that the power law exponent $n$ decreases as the free stream turbulence level is increased, leading to even higher values of the outer region mixing length from equation (31). Thirdly, from the foregoing simplified analysis, the role of the structural coefficient $a_1$ $(-\overline{u'v'} = a_1\bar{q}^2)$ is highlighted. It is clear, for instance, from equation (30), that as $a_1$ is increased so then will the outer region mixing length increase for a given free stream turbulence level. This result is also to be expected on a simple physical basis, since the coefficient $a_1$ represents the "efficiency" of the boundary layer in converting turbulence kinetic energy $\bar{q}^2$ into Reynolds stress $-\overline{u'v'}$.

Finally, a relationship such as equation (28) could serve to correct a simple algebraic mixing length or eddy kinematic viscosity formulation for free stream turbulence effects. However, having proceeded to that stage of complexity it would be a simple matter to utilize the present turbulence model in its entirety. All of the subsequent calculations have been performed using the complete model described earlier.

## 3. COMPARISONS WITH EXPERIMENT

The available experimental data on the effect of free stream turbulence on the turbulent boundary layer falls into two broad categories, that taken primarily for heat-transfer purposes and that taken primarily for aerodynamic purposes. The available heat-transfer information has been summarized by Kestin [9] and, with the exception of the data of Sugawara *et al.* [17] fairly consistently indicates that apart from a movement of the transition point there is little apparent effect on heat transfer due to free stream turbulence. The discordant results of Sugawara *et al.* [17] have been called into question by various authors as a result of their use of a then novel transient measuring technique. In addition, it is observed that Sugawara *et al.* did not reproduce the usual Blasius slope for the Nusselt number vs Reynolds number variation and, furthermore, the transition Reynolds numbers obtained by Sugawara *et al.* were extremely low; lower than could reasonably be obtained without a very powerful tripping agent. In view of these considerations the data of Sugawara *et al.* are not felt to be representative and will not be considered further.

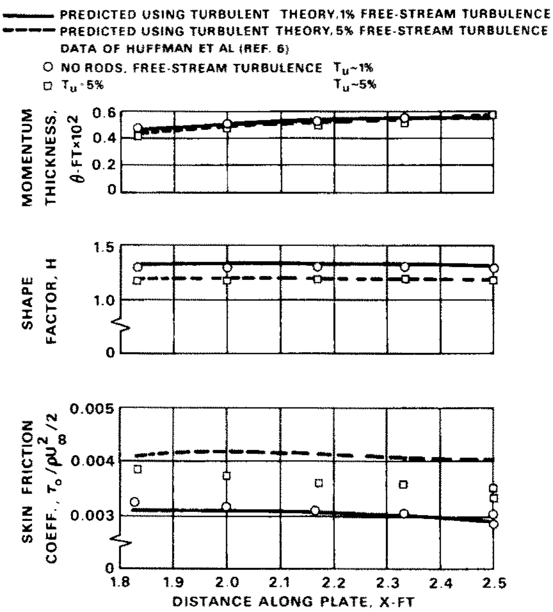


FIG. 1. Effect of free stream turbulence on the boundary layer.

The available aerodynamic data consists mainly of mean velocity profiles for varying free stream turbulence levels. However, Huffman *et al.* [6] and Charnay *et al.* [7] made the extremely valuable additional measurements of Reynolds stress profiles across the boundary layer. The data is all very consistent. Kline *et al.* [4], Schilichting and Das [5], Huffman *et al.* [6], and Charnay *et al.* [7], all found similar effects which, as a result of the stress measurements of Huffman *et al.* and Charnay *et al.* can be attributed to a marked increase in turbulent transport within the boundary layer due to free stream turbulence. The measurements of Kline *et al.* [4] and the measurements presented by Schilichting and Das [5] overlap to some

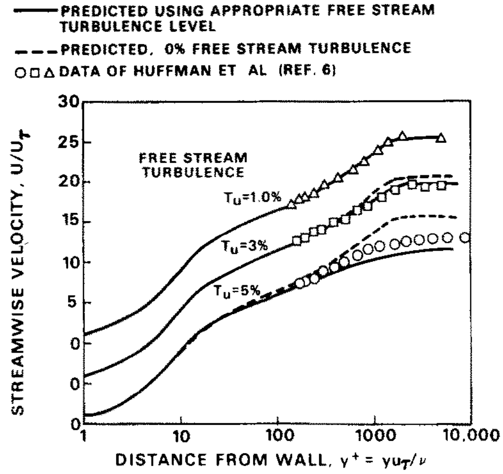


FIG. 2. Effect of free stream turbulence on the boundary-layer mean velocity profile.

extent in terms of Reynolds number range and free stream turbulence level so only the comparisons with Kline *et al.* are presented here. Schilichting and Das did present some data taken in a pressure gradient, but not in sufficient detail to enable a comparison with theory to be made. The measurements of Huffman *et al.* [6] were made on a flat plate downstream of an array of rods. Data for three free stream turbulence levels of approximately 1, 3 and 5 per cent are presented. Evidently, the free stream turbulence was anisotropic and the total turbulence intensity $\bar{q}_e^2/U_e^2$ was somewhat larger than $3T_u^2$ and the calculations were performed using the measured values of $\bar{q}_e^2/U_e^2$. The comparisons between measurements and predictions are shown in Figs. 1–4. In Fig. 1 the usual boundary-layer parameters, momentum thickness $\theta$, shape parameter $H$, and skin friction coefficient are compared for a nominal

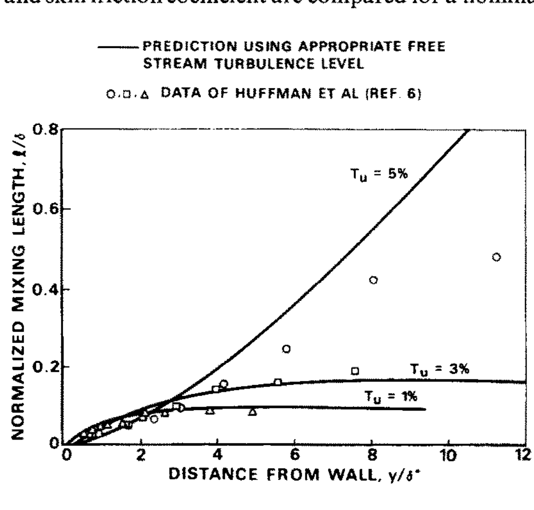


FIG. 3. Variation of Prandtl's mixing length across the boundary layer.

free stream turbulence level of 1 and 5 per cent. The integrated parameters $\theta$ and $H$ show only a slight effect of free stream turbulence and this trend is reproduced by the theory. Skin friction coefficient shows an increase with increasing free stream turbulence level and here the indications are that the theory over predicts the effect slightly. In Fig. 2 velocity profiles are presented in the usual law of the wall coordinates for three turbulence levels. Quite a large effect on the wake component of the boundary layer is observed and once again the predictions are in good agreement with the data, although at the highest turbulence level the theory once again over predicts the observed effects. In Fig. 3 profiles of Prandtl's mixing length across the boundary are shown. In going from 3 to 5 per cent both the predictions and measurements show a very dramatic increase in mixing length in the outer region of the boundary layer indicating a very substantial increase in the turbulent transport due to the free stream turbulence. In Figs. 4 and 5 profiles of the turbulence kinetic energy Reynolds shear stress across the boundary are shown for the three free stream turbulence levels. Once again very large increases are observed and predicted in going from 3 to 5 per cent free stream turbulence. It is clear from Figs. 3, 4 and 5 that the theory does very well in predicting the observed increase in mixing length and turbulence kinetic energy and Reynolds shear stress. Perhaps not surprisingly there were substantial differences between the measured and predicted velocity profile thickness, $\delta_{0\cdot99}$, so to eliminate these differences the profiles were normalized by the displacement thickness.

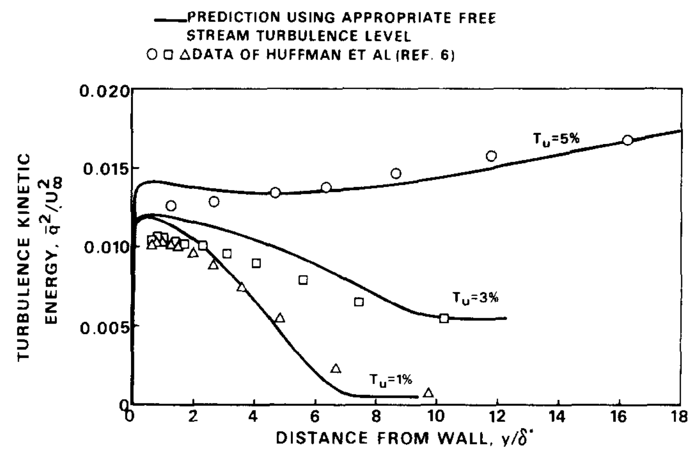


FIG. 4. Variation of turbulence kinetic energy across the boundary layer.

The measurements of Kline, Lisin and Waitman [4], were also made on a flat plate downstream of an array of rods. Kline *et al.* presents velocity profiles at three streamwise locations for rod diameters of 1/8, 3/8, 1/2 and 3/4 in and for no rods. The measured decay of the turbulence downstream of the rods was quite scattered but roughly followed the decay law, $T_u = 1{\cdot}12(x/b)^{-5/7}$, where $b$ is the rod diameter, noted by Baines and Peterson [18]. For consistency in performing the calculations the Baines and Peterson decay law was utilized. The comparisons between measured and predicted velocity profiles for no rods, and for the 1/2 and 3/4 in rods are given in Figs. 6–8. In Huffman *et al.*'s case the computed boundary layer was initiated from the first measuring station using the measured data to start the calculation. In the case of Kline *et al.* in view of the limited number of survey stations the calculation was initiated 1 in downstream from the leading edge of the plate so that by the time the first measuring station was reached the boundary layer was quite insensitive to starting assumptions, other than that the boundary layer was turbulent at the start. As with the

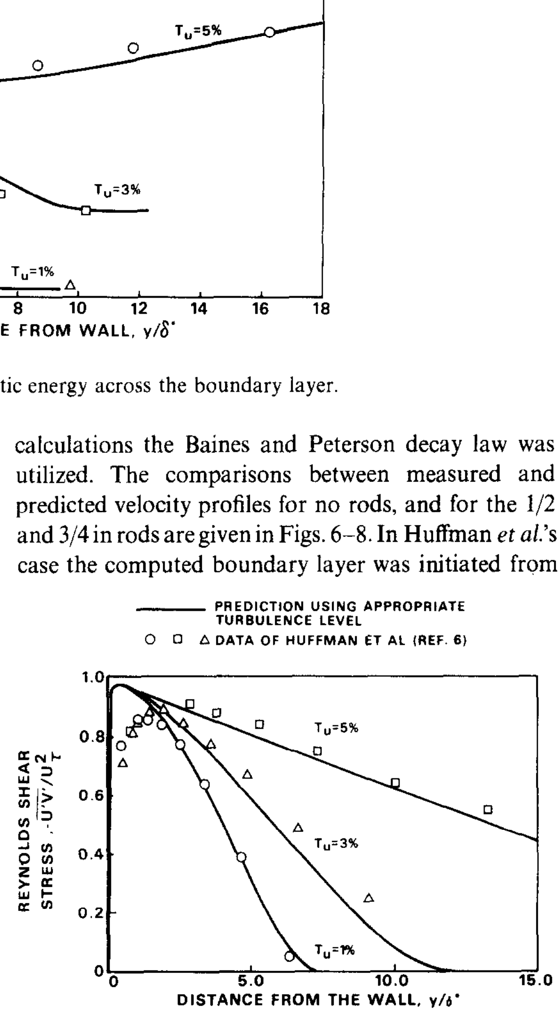


FIG. 5. Variation of Reynolds shear stress across the boundary layer.

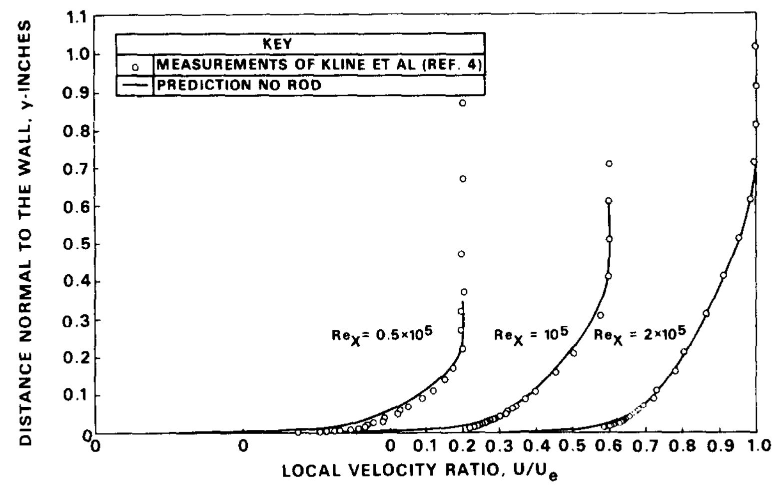


FIG. 6. Effect of free stream turbulence on the boundary-layer mean velocity profile.

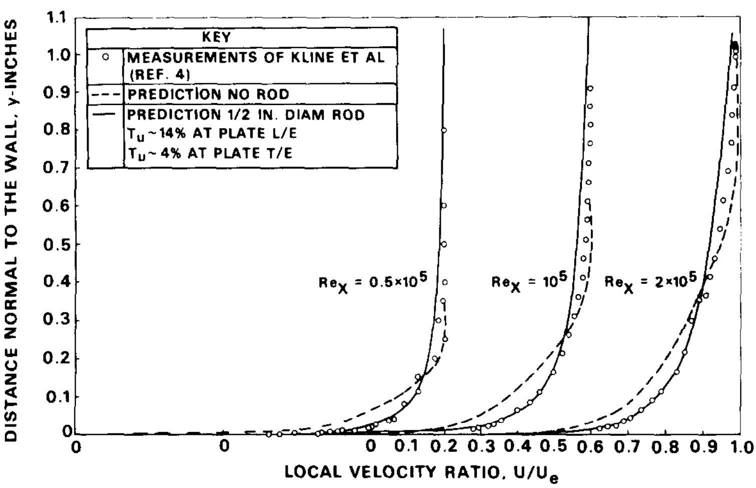


FIG. 7. Effect of free stream turbulence on the boundary-layer mean velocity profile.

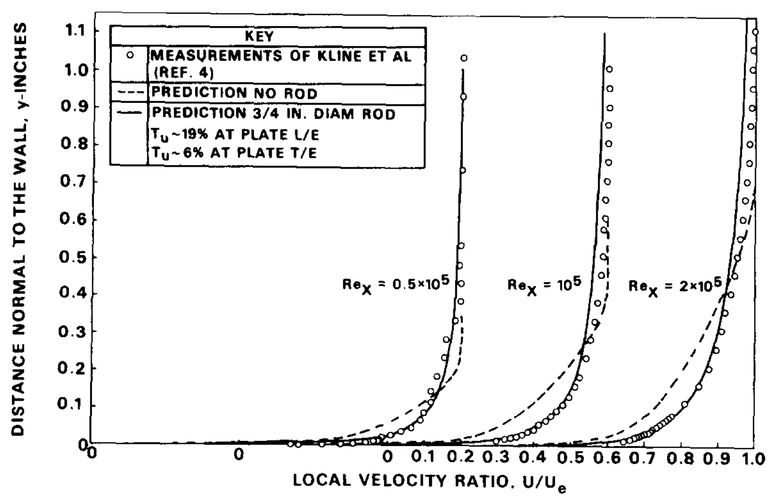


FIG. 8. Effect of free stream turbulence on the boundary-layer mean velocity profile.

comparisons of Huffman *et al.*'s data, the predictions are in very good agreement with the measurements with the observation that perhaps at high free stream turbulence levels the theory over predicts the observed results somewhat.

Charnay *et al.* [7] made some detailed measurements downstream of biplane grids and found very similar effects to those observed by Huffman *et al.* [6] and Kline *et al.* [4]. In addition, however, Charnay *et al.* found that the intensity of the free stream turbulence and not its scale was the important characterizing parameter, in keeping with the present analysis. Charnay *et al.* did measure the effect of free stream turbulence on wall skin friction by means of a Preston tube. In their very similar study considered previously, Huffman *et al.* fitted their mean velocity profile to the law of the wall to obtain measured wall skin frictions and, in view of the scarcity of data points very close to the wall, this process is not so precise as a Preston tube measurement. The comparison between measured and predicted skin friction for Charnay *et al.*'s tests is shown in Fig. 9, and here the agreement is better

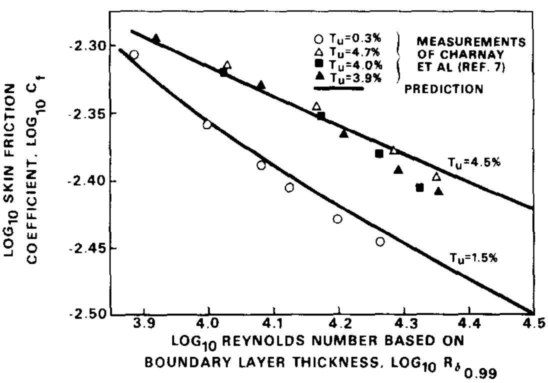


FIG. 9. Effect of free stream turbulence on skin friction.

than with the Huffman *et al.*'s skin friction data. Of particular note is the fact that both theory and experiment indicate that for a constant free stream turbulence level the effect on wall stress is reduced as the Reynolds number is reduced. This observation would seem to have a bearing on the findings of most of the investigators who have examined the effect of free stream turbulence upon heat transfer. One would expect some form of Reynolds analogy to hold and that the heat transfer would be proportional to the skin friction. Thus one would be led to expect less effect of free stream turbulence on heat transfer at low Reynolds numbers. Consequently, the anomalous observation that free stream turbulence has little direct effect on heat transfer may be due to the low Reynolds numbers at which the studies have been performed. Indeed, it turns out that most of the recent studies on the effect of free stream turbulence on heat transfer have concentrated on demonstrating the upstream shift of the transition region with increasing free stream turbulence level and as a result these studies have nearly all been carried out at very low Reynolds numbers. The physical explanation for the decrease in the effect of free stream turbulence on skin friction at low Reynolds number may be as a result of the increased sublayer thickness.

Predictions of the combined effect of free stream turbulence on moving transition and increasing turbulent transport have been made and compared with the data of Kestin *et al.* [9]. These comparisons are presented in Fig. 10 but, in view of the lack of information required to start the calculations, the

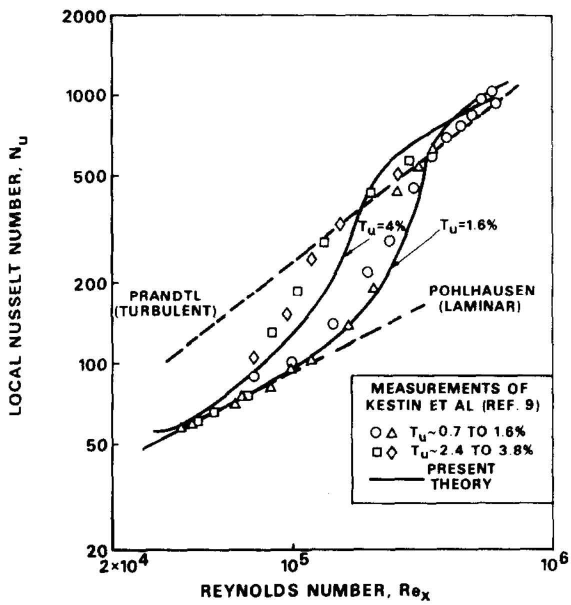


FIG. 10. Heat transfer from a flat plate with varying free stream turbulence.

results are really only illustrative, since starting values for Reynolds stress profiles, for instance, were chosen to give the desired transition locations. Immediately following transition the calculations reproduce the experimental observation of little direct effect of free stream turbulence on the heat transfer. This lack of effect is not entirely inconsistent with increased transport within the boundary layer, since with the higher turbulence level the boundary layer with the early transition is, in fact, thicker at a given streamwise location. Normally, the thicker boundary layer would be expected to have a reduced heat transfer but, since the observer heat transfer is very close to the level for the thinner boundary layer, it could be argued that the transport within the thicker boundary layer must be higher. Lastly, it is observed that profiles of Kline *et al.* although also at low Reynolds numbers, do show a marked effect of free stream turbulence even very close to the wall. However, the level of free stream turbulence found to effect the near wall flow in the Kline *et al.* study was extremely high near the plate leading edge, higher than found in any heat-transfer study performed to date. The calculations indicated that flow at the trailing edge was still exhibiting the effect of the very high levels at the plate leading edge. Thus it is felt that, although Kline *et al.*'s, measurements indicate a wall effect of free stream turbulence even at low Reynolds numbers, in view of the high turbulence levels involved, Kline *et al.*'s measurements are not really in conflict with the idea of a reduction in the effect of free stream turbulence at low Reynolds numbers.

## 4. CONCLUSIONS

The aim of the present study was, firstly, to see if the theory of McDonald and Fish could provide the framework for predicting the observed large effects of free stream turbulence upon the turbulent boundary layer and, secondly, to see if some explanation could be found for the lack of effect of free stream turbulence on heat transfer. On the basis of the comparisons between predictions and measurements it is clear that by allowing for the entrainment of free stream turbulence into the boundary layer and performing a turbulence energy balance in the manner used by McDonald and Fish, very satisfactory predictions of the effects of free stream turbulence can be made. Insofar as the observed lack of effect of free stream turbulence on heat transfer is concerned, this may be only superficial and, in any case, restricted to low Reynolds numbers, Certainly, if Reynolds analogy holds (and the theory indicates that it does, although this to some extent is built in as a result of assuming a turbulent Prandtl number), 30 per cent increases in heat transfer can be expected with turbulence levels around 5 per cent at Reynolds numbers based on momentum thickness of five thousand or more. Possibly as a result of the thickening of the sublayer, the effect of free stream turbulence on heat transfer is lessened at low Reynolds numbers, and this is where most of the experimental heat-transfer studies have been performed to date. Even in the low Reynolds number studies the expected decrease in Nusselt number due to an upstream shift in virtual origin of the turbulent boundary layer with increasing free stream turbulence level, is not observed; indicating that even at low Reynolds number the free stream turbulence is augmenting the transport within the boundary layer.

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## EFFET DE LA TURBULENCE DE L'ECOULEMENT LIBRE SUR LA COUCHE LIMITE TURBULENTE

**Résumé**—On montre dans cette étude qu'en supposant la pénétration de la turbulence de l'écoulement libre dans la couche limite turbulente et qu'en exprimant un bilan global d'énergie turbulente, on peut établir des estimations quantitatives très satisfaisantes de l'effet de la turbulence de l'écoulement libre sur le comportement de la couche limite turbulente. La théorie et l'expérience indiquent une augmentation de 30 pour cent du frottement pariétal, pour un niveau de turbulence libre égal à 5 pour cent seulement. Puisqu'il semble exister une sorte d'analogie de Reynolds, on suggère que le résultat fréquemment obtenu d'un effet direct faible du niveau de turbulence libre sur le transfert thermique, par autre chose que le déplacement du point de transition, peut être valable seulement aux faibles nombres de Reynolds.

## DIE AUSWIRKUNG DER TURBULENZ DER FREIEN STRÖMUNG AUF DIE TURBULENTE GRENSCHICHT

**Zusammenfassung**—Bringt man ein Eindringen der Turbulenz der freien Strömung in die turbulente Grenzschicht in Ansatz und zieht eine vollständige Energiebilanz der Turbulenz, so kann man, wie die vorliegende Untersuchung zeigt, sehr befriedigende Voraussagen über die Auswirkung der Turbulenz der freien Strömung auf das Verhalten der turbulenten Grenzschicht machen. In Theorie und Experiment läßt sich eine 30prozentige Zunahme der Wandreibung bei einem Turbulenzgrad von nur 5% nachweisen. Da eine gewisse Reynolds-Analogie wohl gegeben sein dürfte, wird vorgeschlagen, den häufig zitierten Befund eines geringen, direkten Einflusses des Turbulenzgrads der freien Strömung auf die Wärmeübertragung—im Unterschied zu der Verschiebung des Umschlagpunkts—nur bei niedrigen Reynolds-Zahlen als gültig zu betrachten.

## ВЛИЯНИЕ ТУРБУЛЕНТНОСТИ СВОБОДНОГО ПОТОКА НА ТУРБУЛЕНТНЫЙ ПОГРАНИЧНЫЙ СЛОЙ

**Аннотация** — В данном исследовании отмечается, что, учитывая проникновение турбулентности свободного потока в турбулентный пограничный слой при условии выполнения общего баланса турбулентной энергии, можно весьма удовлетворительно количественно предсказать влияние турбулентности свободного потока на поведение турбулентного пограничного слоя. Более того, как теория, так и эксперимент указывают на 30%-ное увеличение трения на стенке при изменении степени турбулентности свободного потока только на 5%. Так как можно предположить, что здесь имеет место что-то вроде аналогии Рейнольдса, то приводимый в литературе факт незначительного влияния степени турбулентности свободного потока непосредственно на теплообмен, а не на движение точки перехода, может иметь место при малых числах Рейнольдса.